Unit 3: Probability

Differentiated Instruction Teaching/Learning Examples

Duration: Determined by Teacher
1 Group Discussion—Chart
2 Tally Charts
3 Communicating Understanding
4 Reflection

Differentiated Instruction Details

KNOWLEDGE OF STUDENTS

Differentiation based on student:

- ☑ Readiness
- ☑ Interests
- ☑ Preferences: ☑ Styles ☑ Intelligences ☐ Other (e.g., environment, gender, culture)

NEED TO KNOW

- Students' preferred way of learning in order to support the in-class sessions and their RAFT choices

HOW TO FIND OUT

- Self-assessment based on the multiple intelligences

DIFFERENTIATED INSTRUCTION RESPONSE

- ☑ Learning materials (content)
- ☑ Ways of learning (process)
- ☑ Ways of demonstrating learning (product)
- ☑ Learning environment

Assessment and Evaluation

ASSESSMENT/SUCCESS CRITERIA

Thinking

- Demonstrates reasoning and proving based on data given
- Demonstrates an understanding of skills to extract and interpret graph and table information
- Demonstrates an understanding of a variety of graphs and charts

Communication

- Communicates in descriptive terms the meaning of the data and the relevance to the subject

Application

- Makes connections to real world situations that use data for evidence
- Uses appropriate graphic organizer or visual to present findings

Assessment Tools

> Rubric
> Checklists

Prior Learning

Prior to this lesson, students will have:

- Awareness of graphs and some probability techniques
- Awareness of collaborative processes
- Awareness of the inquiry process
- Awareness of forms of gambling

Materials and Resources

MATERIALS

Appendix A

- Student Resource 1: Lottery Scenario
- Student Resource 2: Tally Sheets
- Student Resource 3: Randomness
- Student Resource 4: Coins, Dice and Cards
- Student Resource 5: Recognition of Patterns
- Student Resource 6: Coin Flip
- Student Resource 7: The 3 Rs of Reflection

Appendix B

- Teacher Resource 1: Properties of Probability
- Teacher Resource 2: Randomness
- Teacher Resource 3: Coins, Dice and Cards

INTERNET RESOURCES

Ontario Lottery and Gaming Corporation (OLG)
Know your limit. Play within it.
www.knowyourlimit.ca/
KnowYourLimit.ca is a web-based resource that provides information about how gambling works in Ontario, myths and facts, game odds and helpful tips to keep gambling fun. The link above is for a video called The Slot Machine: What Every Player Needs to Know that explains the concepts of odds and randomness in a visual way.

Addictions Foundation of Manitoba
Cost of Play Calculator
http://getgamblingfacts.ca/cop/english/cop.html
This interactive website provides facts about gambling to help readers make more informed choices. The link is for the Cost of Play Calculator, an interactive tool that helps individuals learn more about how to reduce the cost of play.

RESOURCES

Dice and Cards

Lesson Plan

Minds On

> **Establishing a positive learning environment**
> **Connecting to prior learning and/or experiences**
> **Setting the context for learning**

Connections: L: Literacy; ML: Mathematical Literacy; AfL, AoL: Assessment for/of Learning; SC: Skill Continua

SMALL GROUPS

- Students are provided with a chart having three sections: "I see," "I think," "And so." Brainstorm a list of "I see" statements to a picture of a gambling scene.
- Students then answer the statement "I think," completing that chart section (What do you think is happening in the story? What does the evidence tell you?).
- Students then complete the "And so" section of the chart (I conclude…, I think that…).

AfL: Chart

WHOLE CLASS DEBRIEF

- Teacher directs the discussion to review inferences and pros and cons of statements made in the activity.

AfL: Anecdotal Comments

Action

> **Introducing new learning or extending/reinforcing prior learning**
> **Providing opportunities for practice and application of learning (guided > independent)**

INDIVIDUAL

- Working independently, students complete the activity in Student Resource 1: Lottery Scenario, which talks about a specific lottery situation.

AfL: Schematic Presentations or Charts, Reflections

WHOLE CLASS DEBRIEF

- Teacher directs discussion to ensure that after this exercise students understand probability, average and guarantee.

Additional information to help in the discussion:

- **Question 3:** It is possible that a person could win every week for a year. The chances are (1/4) to the power of 52 = 1 chance in 20,282, 409,603,651,700,000,000,000,000,000. It is so incredibly unlikely that it is virtually impossible. Nonetheless, although extremely unlikely it is possible.
- **Question 4:** It is possible that person could lose every single draw for a year. The chances of losing are (3/4) to the power of 52 = 1 chance in 3,139,166. This means that losing every single draw is also very unlikely; but much less unlikely that winning every draw.

It is important to understand the difference between something that is impossible, something virtually impossible (e.g., winning every bet for a year) and something that is very unlikely (e.g., losing every bet for a year).

AfL: Anecdotal Comments

Connections

A person who experiences a large win gambling may think that their success is so unlikely that it could not have been chance and may attribute their win to the concept of luck, prayer, a betting system, or skill. Unusual wins and unusual losses can lead people to distorted beliefs about their ability to win.

SMALL GROUPS

- There are three small group activities to select. Depending on the amount of time the teacher has available, one or all of the activities may be selected. The lesson activities are:

1. **Properties of Probability**
   The Properties of Probability involves a variety of tally chart activities. The more variety in the activities, the greater the increase in student understanding of the concept of probability and house edge. Students must work through each activity for the recommended number of times to see long-term outcomes. This activity illustrates several properties of probability and house edge. Teacher Resource 1 provides the instructions and Student Resource 2 provides the tally sheet and worksheet for the students to complete the activities. The activities are divided into eight separate sections that must be completed in the order presented. Sections 1 and 2 look at probability and sections 3 to 8 look at house edge.
2. **Randomness**
   The Randomness activity takes a closer look at what randomness is and how it may impact beliefs about chances of winning when involved in some gambling activities. Teacher Resource 2 and Student Resource 3 provide the instructions and handouts.
3. **Coins, Dice and Cards**
   Coins, Dice and Cards are activities with coins, dice and cards that teach about probability. Teacher Resource 3, and Students resources 4, 5 and 6 are used for this activity.

WHOLE CLASS DEBRIEF

- Students reflect on the results and should demonstrate an understanding of sample size, properties of probability, range or spread of outcomes, uncertainty, feelings about winning or losing in controlled circumstances, randomness, chance of winning and random events.

AfL: Anecdotal Comments

Consolidation and Connection

> **Helping students demonstrate what they have learned**
> **Providing opportunities for consolidation and reflection**

INDIVIDUAL RAFT

- The RAFT assignments are differentiated based on student intelligence preferences. RAFT assignments can be done independently, or in pairs or groups in class or out of class. In all cases students require an understanding of the aspects of a RAFT topic, and rubrics need to be supplied for all topic possibilities.

Connections

DIFFERENTIATED INSTRUCTION TEACHING/LEARNING EXAMPLES

	Role	Audience	Format	Topic
1	Teacher	Young children	Graphs, etc.	Plan a series of probability experiments for students in grades 3 to 5 to teach them what you have learned in this session.
2	Health educator	Adult	Game evaluation	Evaluate a number of child and youth games to determine the amount of gambling that is part of the game. Present your material in a report you will give to the parent council of a senior public school.
3	Teacher	Students	Terminology fact sheet	Prepare a fact sheet with terminology from this session and multiple examples to explain each term.
4	Friend	Youth	Personal e-mail	Write an e-mail to a friend whom you now suspect, after what you have learned in this unit, is much deeper into gambling activities than you thought. Express your concern and the reasons for that concern.
5	Group counsellor	Adults	Business letter	Watching and listening to your after-school group play and converse with each other, you are concerned about their talk about evening computer activities: pool, hearts, euchre, blackjack, etc., which involve a level of gambling. Prepare a proposal to do further research into this matter.
6	Self	Adult	Survey	You have listened to a presentation on gambling and you suspect that your activities include a number of gambling activities. Keep a record for a week of the games you play and record your result with summation in an appropriate manner.
7	Teacher, student helper	Adult	Conversation	You are a student helper for a teacher. You admire the professionalism of the teacher but you are concerned that this person may be seriously involved in casino activities. You are fearful of serious consequences for this teacher. Write the conversation that you will have with this individual to try to see if your thoughts are correct, and also to judge the teacher's willingness to seek help.

Connections

	Role	Audience	Format	Topic
8	Researcher	Adult	Television program assessment	Select a program on television that you suspect is a form of gambling. Watch the program three times. Record incidents of "gambling." Indicate when it happens, what the result is and whether it could lead to a problem for the contestants and audience.

INDIVIDUAL METACOGNITION

- The 3 Rs of Reflection: Students use The 3 Rs of Reflection (Retell, Relate, Reflect) as a way for them to reflect on their learning prior to the next class.

Student Resource 1

Lottery Scenario

SCENARIO

Imagine that there is a lottery where one winner is picked once a week for one year. There are only four tickets in the lottery and you purchase one for yourself every week of the entire year (52 weeks). What is your probability of winning expressed as a per cent?

PROCEDURE

1. Create a T-chart in your notes to tally the results of your investigation. The T-chart will have 52 weeks listed in the left-hand column and your answers in the right hand column.
2. Create four tickets, each numbered with a different number.
3. Place the tickets in a container, and without looking pull one ticket for each week of the year. As you pull the tickets, record the number of the ticket beside the week it represents.
4. Calculate the number of times that each ticket won.
5. Change this number into a per cent.
6. Answer the following questions.

DISCUSSIONS

1. In the first week what is your probability of winning?

2. Is it possible to create a formula that represents your chance of winning each week?

3. Is it true that you could win every week all year?

4. Is it true that you might not win in any week?

5. Why is there no guarantee that you might win?

Student Resource 2 P. 1

Tally Sheet

SECTION ONE
Individual Tally Sheets

	Your Choice	Actual Throw	Win	Lose
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				
		Totals		

Student Resource 2 P. 2

SECTION TWO
Group Tally Sheet

	Win	Lose
Person 1	_____	_____
Person 2	_____	_____
Person 3	_____	_____
Person 4	_____	_____
Person 5	_____	_____
Totals	_____	_____

SECTION THREE
Individual Results

Per cent of wins after 20 throws (for a group of 5 people)

Probability = wins ÷ 20 x 100 = _____

Probability = _____ ÷ 20 x 100 = _____ %

Per cent wins after 25 throws (for a group of 4 people)

Probability = wins ÷ 25 x 100 = _____

Probability = _____ ÷ 25 x 100 = _____ %

SECTION FOUR
Group results (assuming 100 rolls of the dice in total)

Probability = wins ÷ 100 x 100 = _____

Probability = _____ ÷ 100 x 100 = _____ %

SECTION FIVE
No house edge (prize = 6)

Individual per cent win from Section Three = _____

House edge = 100 – (percent win) x 6

House edge = 100 – (_____) x 6 = _____

Student Resource 2 P. 3

Group per cent win from Section Four = _____

House edge = 100 – (percent win) x 6

House edge = 100 – (_____) x 6 = _____

SECTION SIX

Moderate house edge (prize = 5)

Individual per cent win from Section Three = _____

House edge = 100 – (percent win) x 5

House edge = 100 – (_____) x 5 = _____

Group per cent win from Section Four = _____

House edge = 100 – (percent win) x 5

House edge = 100 – (_____) x 5 = _____

SECTION SEVEN

Huge house edge (prize = 4)

Individual percent win from Section Three = _____

House edge = 100 – (percent win) x 4

House edge = 100 – (_____) x 4 = _____

Group per cent win from Section Four = _____

House edge = 100 – (percent win) x 4

House edge = 100 – (_____) x 4 = _____

SECTION EIGHT

Negative house edge (prize = 7)

Individual percent win from Section Three = _____

House edge = 100 – (percent win) x 7

House edge = 100 – (_____) x 7 = _____

Group percent win from Section Four = _____

House edge = 100 – (percent win) x 7

House edge = 100 – (_____) x 7 = _____

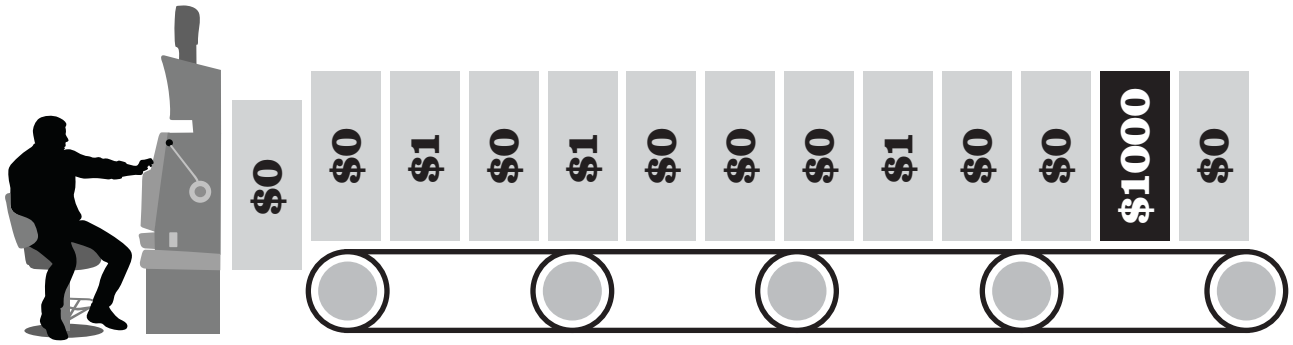
Student Resource 3 P. 1

Randomness

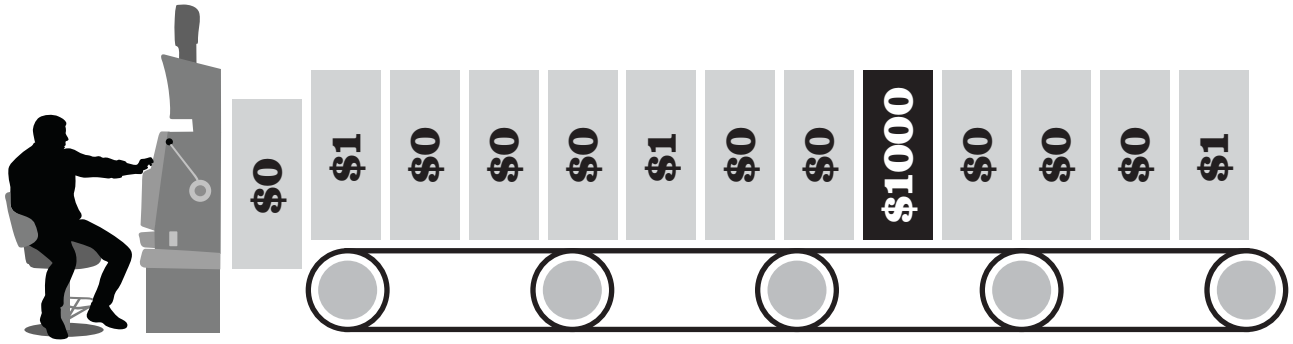
FIGURE 1: This is what people think random chance is like.

The Conveyor Belt Myth: The Dream

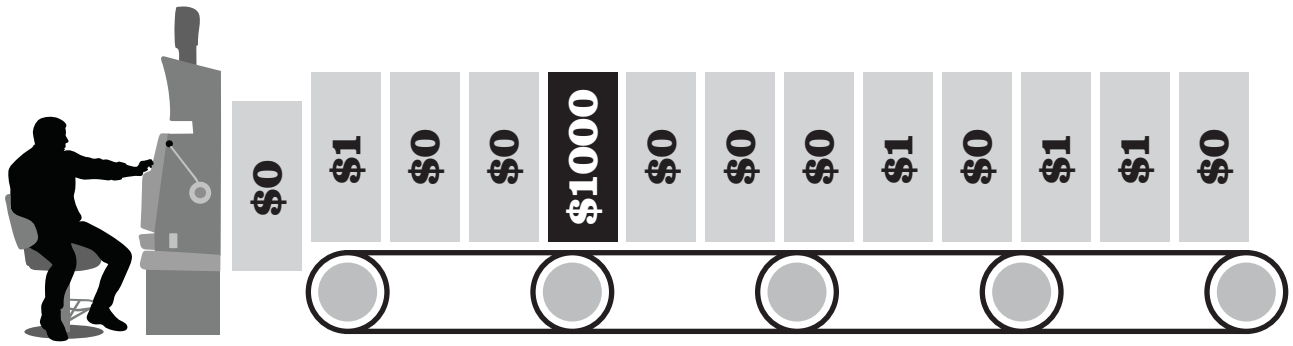
Play begins. The win is a long way off.



After a few hours, the big win is a little closer.



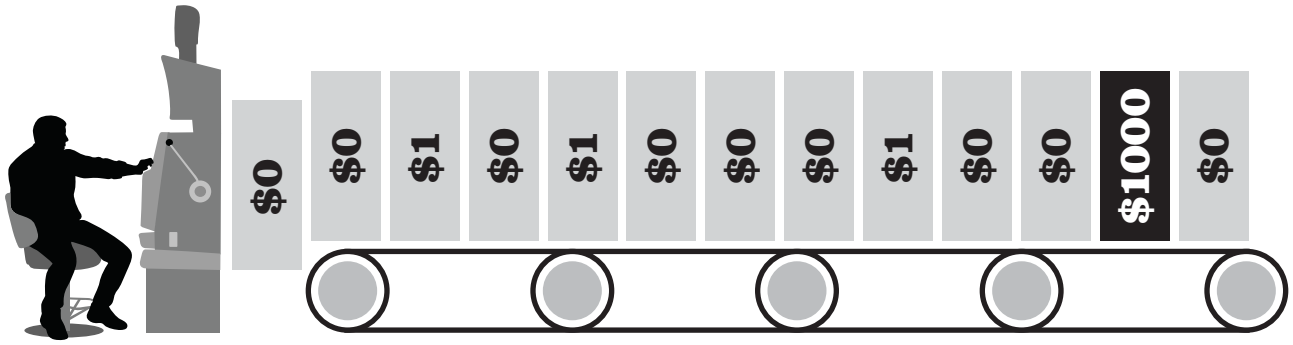
As time goes by, the win gets closer and closer.



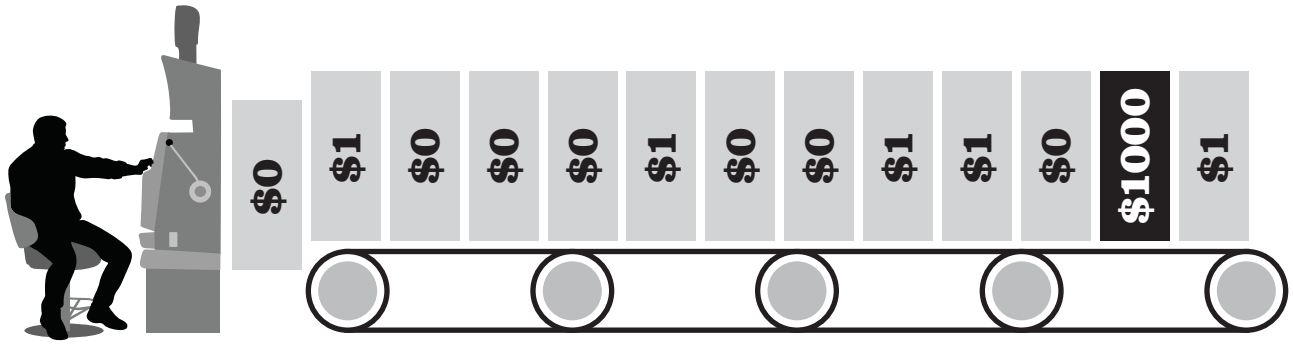
Student Resource 3 P. 2

The Conveyor Belt Myth: The Reality

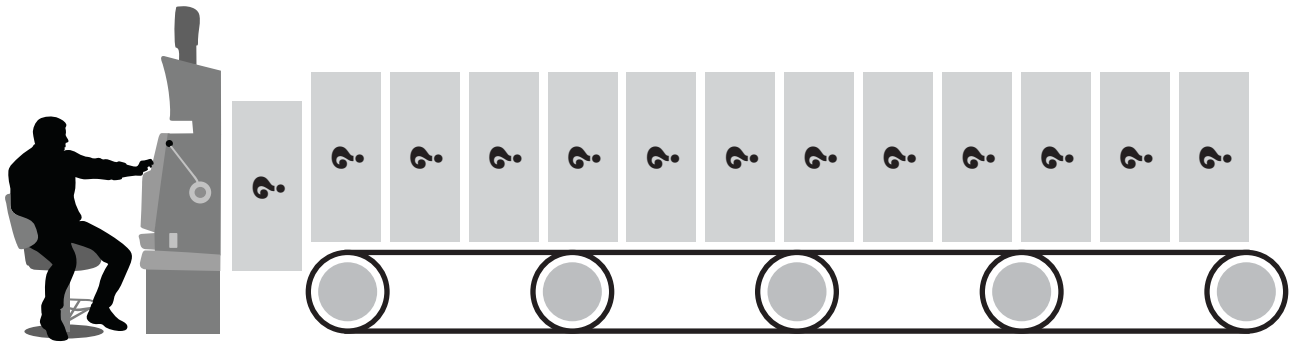
Play begins. The win is a long way off.



There is no conveyor belt. The odds do not improve over time.



You never know if you are going to win or lose.



The only thing you know for sure is that you are unlikely to win a large prize.

Student Resource 4 P. 1

Coins, Dice and Cards

QUESTION 1

Flip a coin 20 times. Record your results in the table below.

a) How many heads are there?

b) How many tails are there?

c) Can you use this information to predict how many heads and tails will occur in the next 20 coin flips? Explain your thinking.

d) Flip the coin 20 more times. Record your results in the table below.

e) Was the number of heads and tails the same as your first result? Does this support or refute your answer to question “c” above? Explain your thinking.

Student Resource 4 P. 2

QUESTION 2

a) Now, suppose that the coin has already been flipped 14 times and the results are as follows: 12 heads and 2 tails. What will happen next? Flip a coin 36 more times until you have a total of 50 flips. Does the number of heads and tails even up?

Heads	𝍸 𝍸 ||	12 + _____ = _____
Tails	||	2 + _____ = _____

Did the number of tails catch up to the number of heads? Yes No

What percentage of tosses is heads?

What percentage of tosses is tails?

Would you expect the number of tails catch up to the number of heads eventually? Why or why not?

Student Resource 4 P. 3

b) Now, suppose that the coin has already been flipped 414 times and the results are as follows: 212 heads and 202 tails. What will happen next? Flip a coin 36 more times until you have a total of 450 flips. Does the number of heads and tails even up?

Heads		212 + _____ = _____
Tails		202 + _____ = _____

Did the number of tails catch up to the number of heads? Yes No

What percentage of tosses is heads?

What percentage of tosses is tails?

Would you expect the number of tails catch up to the number of heads eventually? Why or why not?

QUESTION 3

Compare your conclusions from questions 2a with those from 2b. Compare the actual number of heads and tails in each experiment. Now compare the percentage of heads and tails in each.

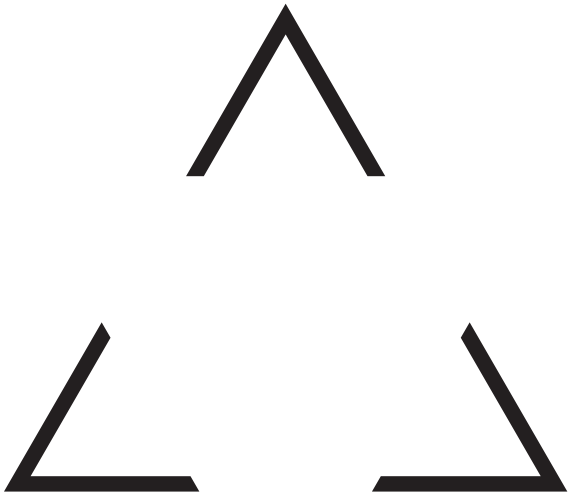
Student Resource 5 P. 1

Recognition of Patterns

What do you see?

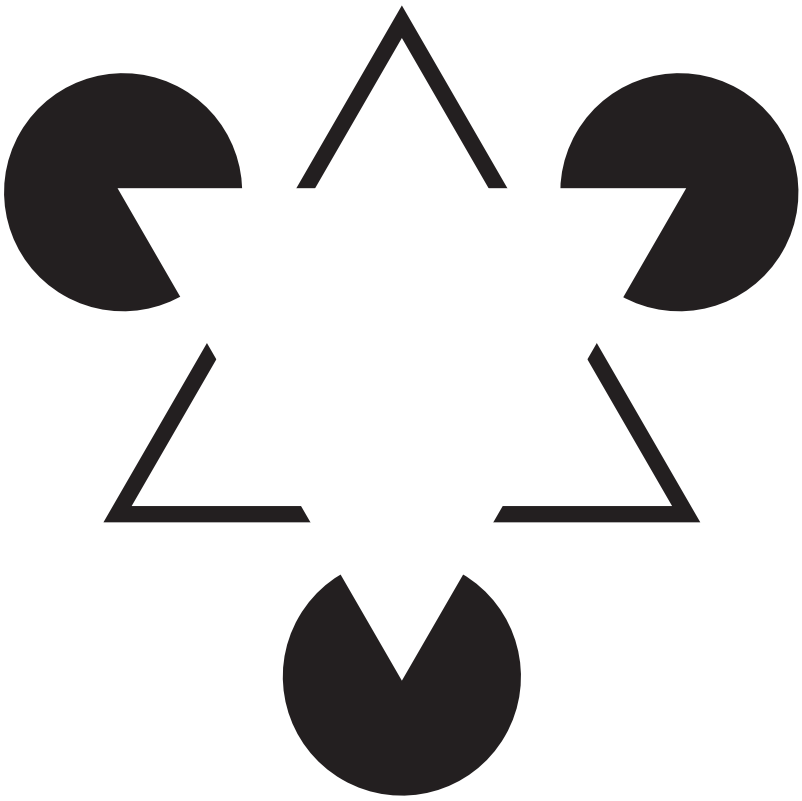
What do you see?

What do you see?



Student Resource 5 P. 2

Although there are no actual triangles that appear on your eyes' retinas, your brain will somehow interpret the following image as two overlapping triangles. Is this imagination? Are you losing your mind? No. The notched circles and angled lines merely suggest gaps in which objects should be. The brain does the rest by triggering a sort of pattern recognition phenomenon.

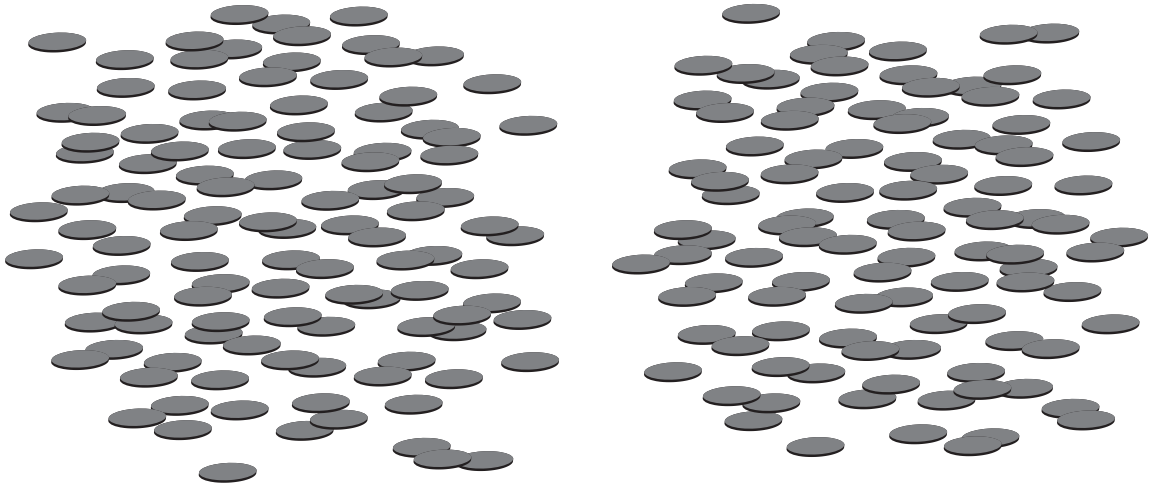


Student Resource 6

Coin Flip

After 14 flips you have 12 heads and 2 tails

After 414 flips you have 212 heads and 202 tails



Student Resource 7

The 3 Rs of Reflection

NON-NEGOTIABLE

Use the 3 Rs (Retell, Relate, Reflect) to explain your learning.

NEGOTIABLE

1. Topics

Select one of the topics below for your reflection:

a) Skills and knowledge that I acquired
b) The importance of following through the entire process of an experiment
c) My comfort level in working in small groups or with a partner
d) I can apply the following information to my life

2. Formats

Select one of the following presentation formats for your reflection:

a) Written reflection—point form or paragraph
b) Recorded oral reflection (mini recorder)
c) Presentation software—slides with images and graphics
d) Blog

OPTIONAL

A. Make a two-minute presentation of reflective learning during a class.
B. Act as a guest speaker in the class.

Teacher Resource 1 P. 1

Properties of Probability

INTRODUCTION TO SECTIONS ONE THROUGH FOUR (see below)

This activity illustrates several properties of probability:

1. Probability is completely uncertain on any specific roll—you never know what will happen next.
2. As the sample size increases, the spread of outcomes comes closer to the long-term expected probability.
3. However, even large samples have some uncertainty.
4. You may find traces of odd occurrences or patterns, e.g., two, three or four wins in a row, sequences in the outcomes (1, 2, 3, 4), repeated numbers (4, 4, 4) and patterns (3, 6, 3, 6).

PROCEDURE FOR SECTIONS ONE AND TWO

Have the students work in groups of four or five. Each student is given a die and selects a number from 1 to 6 to bet on. Students can bet on the same number if they wish, but encourage them to select different numbers. They then roll the dice and record the number of each roll of each die on the Tally Sheet. If they are in a group of four, each student should roll his or her die 25 times. If they are a group of five, each student should roll his or her die 20 times so the combined total of dice throws is 100. Tally the wins of the group in Section Two.

PROCEDURE FOR SECTIONS THREE AND FOUR

Next, students need to determine how often they win. After 20 or 25 throws (depending on number of students in group) of the dice, divide the total number of times their number came up by the total number of throws. For example: probability = wins ÷ throws. The formula for the group is probability = wins ÷ 100 x 100. The "÷ 100 x 100" is unnecessary if you have exactly 100 throws, but is put here to show that the formula is the same for 20, 25 or 100 throws.

DISCUSSION

1. How close is the number to 16.6%? Write down on the board the students numbers, arranging them from lowest to highest percentage. The theoretical long-term outcome is 16.6%. Which is closer to 16.6%, the individual results or the group results?
2. Find out which student won the most often. Ask how winning most often made the student feel. Does it make you feel really good to win?
3. Find out which student won the least often. Ask how not winning made the student feel.
4. Go over the tally sheets and look for odd occurrences, e.g., two, three or four wins in a row, sequences in the outcomes (1, 2, 3, 4), repeated numbers (4, 4, 4) and patterns (3, 6, 3, 6). These things will not occur very often, but they will occur and illustrate that weird events do happen.

Teacher Resource 1 P. 2

5. Most people believe that after you win you are less likely to win again. Because of this, people sometimes switch numbers, or switch machines after a big win. As students to look at the first number they rolled. Ask how many of them rolled the same number again on the 2nd roll, how many rolled the first number on the 3rd roll and so on. Make a graph of how students repeated the first number on the 2nd, 3rd, 4th, 5th, 6th and 7th rolls of the dice. On average the number should have been repeated on the 2nd, 3rd, 4th, 5th, 6th and 7th rolls of the dice approximately 16.7% of the time. The results should illustrate that after a number has come up it is neither more likely nor less likely to come up again. Ask them how many rolled the same number more than once in the next 6 rolls. About 26% of the class should report getting the same number more than once. Ask did not repeat the same number in the next 6 rolls. About 33% of the class should report that they did not repeat the same number.

   This belief that a number is less likely to come up again can lead some people to believe they can beat the house edge of the game by avoiding numbers that have come up recently or by looking for numbers that are due to come up. This exercise will demonstrate that the odds of a number do not change after a number has come up.

6. Finally, combine the results from all students to see if the total of all the students is closer to 16.6%.

INTRODUCTION TO SECTIONS FIVE THROUGH EIGHT

Next, the students are introduced to the concept of house edge. House edge is the profit the casino makes from people who gamble. In sections Five through Eight, students see how casinos make their money by not paying back enough money to the players to make up for the money the player has lost. Sections Five through Eight demonstrate how the casino can guarantee a profit by varying the amount paid back to the player. The students can work this out with both their individual tally sheets results (wins and losses) and with the group results. The group results will be closer to the true long-term percentages. In Section Five, there is no house edge so the casino would not make any profit.

PROCEDURE

- Using the results from the tally sheets, place the results in sections Five through Eight. Starting in Section Five, students assume a bet size of 1 penny for every roll of the dice and a prize value of 6 pennies for every win. Compute the house edge assuming a prize of 6 pennies per win. This can be computed for both individual and group data.
- As a group, discuss the outcome and house edge. On average there should be a zero house edge because the prize of 6 pennies is large enough to make up for the chances of losing.
- Now repeat this using a win amount of only 5 pennies for a win. A prize of only 5 pennies should result in an outcome of a net loss of 15 pennies and a house edge of 16.7%.
- Now repeat this with a win amount of only 4 pennies for a win. A prize of only 4 pennies should result in an outcome of a net loss of 30 pennies and a house edge of 33.3%.
- Now repeat this with a win amount of 7 pennies for a win. A prize of 7 pennies should result in an outcome of a net win of 15 pennies and a house edge of –16.7% (or a player advantage of 16.7%).
- Which of these games is the most realistic for a commercial casino: a prize of 6, 5, 4 or 7 pennies for a 1-penny bet? The answer is 5. With 6 or 7 the casino would lose money, and with 4 the player would lose too much, bet bored and go home.

Teacher Resource 1 P. 3

DISCUSSION

The house edge in Section Five is 0%, the house edge in Section Six is 16.6%, the house edge in Section Seven is 33.3%, the house edge in Section Eight is -16.7%.

1. Which was closer to the true values, the individual outcomes or the combined group data?
2. Which house edge would you never see in a real casino?
3. Which house edge would you most likely see in a real casino?

ANSWER

1. The group data will, in general, be closer than the individual results to the true values.
2. You would never see the house edge in Section Eight (prize = 7), and would most likely see the house edge shown in Section Six (prize = 5).
3. The moderate house edge guarantees a profit, but does not take the players' money away too fast. In fact an even more realistic payout would be 5.5 for a house edge 8.4%. A smaller house edge would encourage the player to play longer.

TEACHER NOTE

House edge is the profit the casino makes from people who gamble. Stores such as Walmart make money by charging the customer more money for a product than it cost them. For example, it might cost them $10 to buy a shoe from a warehouse, $1 to transport it across the country, and $3 for the labour cost of putting it on the shelf, for a total cost of $14. They sell the shoe for $24, ensuring a profit for the company of $10. Commercial gambling is just another big business. To ensure a profit, commercial games of chance always have a house edge. That is, the company running the business sets up the game in a way to ensure that it will make money in the long run. This is done by paying out less money for a win than the chance against winning. In Section Five, there is no house edge, so the casino would not make any profit

Teacher Resource 2 P. 1

Randomness

INTRODUCTION

Students review the Conveyer Belt found in Student Resource 3, and discuss the concept of randomness. The teacher can also use the video *The Slot Machine: What Every Player Needs to Know*, available at www.knowyourlimit.ca. This video describes the concept of randomness.

CLASS DISCUSSION

The Conveyor Belt Figure 1 illustrates what people often believe random chance is like. They seem to believe that the prizes are on a conveyor belt that is moving forward. With each spin, the possible win moves closer. Figure 2 illustrates a more realistic depiction in which the chance of winning the big prize does not change no matter how often the person has played. A person never knows if he or she is about to win. This is true regardless of the type of gambling, be it slots, dice, cards, horse race bets or sports bets—the simple truth is that no matter what has occurred, a person's chances of winning are exactly the same from bet to bet. Someone could win on the first spin, or not win for hour after hour.

TEACHER NOTES

What does "random" mean?

- Games of chance use random-numbers generators such as dice, shuffling of cards, flipping of coins or computer programs to generate random experiences for the players.
- Random does not mean that something has no cause. Random events are the results of forces such as gravity, wind, rain and temperature, but these forces are combined in a way that makes it impossible to predict what will happen.
- Random events are the result of complexity, which means too many forces are at work to keep track of. For example, think of all the different forces at work when you roll dice (what size the dice are, how hard you throw them, whether you shake them, how level the table is, where they land). Flipping a coin is too simple to serve as a good example of random chance because it is too easy to manipulate. A good magician can learn the skill to time a coin flip to land on the side he wants. A pair of dice is much better because the movement and bounce of the dice is more complex and much harder to manipulate.
- You can compute your chances of winning, but you cannot tell if or when you will win. For example, if you play a lottery, you might hear the phrase "1 in 10 chance of winning." However, this does not mean that if you play 10 times, you will definitely win once.

RANDOM EVENTS ARE INDEPENDENT

- Independence means that one event cannot affect any other.
- The fact that heads have come up 10 times in a row tells you nothing about what will come up next. The chances of a head coming up are the same regardless of what has come up already. Coins and dice have no memory. Even if your coin came up heads 20 times in a row, that would still not tell you which side will come up next.
- You could win all the time, some of the time or not at all, because random events are independent of each other.

Teacher Resource 2 P. 2

- Some people believe that if they have lost a lot they are "due for a win." When mixed up with emotions and "feeling lucky," this misunderstanding can get people into a lot of trouble. You are never "due for a win."

Teacher Resource 3 P. 1

Coins, Dice and Cards

INTRODUCTION

Provide the students with copies of the Student Resource 4: Coins, Dice and Cards. Ask them to answer questions 1 and 2. Coins and dice should be available for students to use.

Question 1a to e gives students first-hand experience with coins and random chance. It is designed to show students that they cannot predict what will happen next. If they get more heads than tail during the first 20 flips, they may not necessarily get more tail then heads during the next 20 flips.

Question 2a, the students are told that the coin has already been flipped 14 times and they are asked to flip the coin an additional 36 times to equal 50 flips in total. Question 2a is designed to illustrate the fact that the number of heads and tails may not even up.

Question 2b, the students are told that the coin has already been flipped 414 times and they are asked to flip the coin an addition 36 times to equal 450 flips in total. Question 2b is designed to illustrate the fact that even though the actual number of flips may not even up, the percentage of heads and tails will be much close to 50% than in question 2a. The fact that the percentage converges towards 50% creates and illusion that random chance corrects itself. The reality is that the difference of 10 more heads than tails becomes proportionally smaller as the sample size increases, but the number of heads and tails does not actually even up.

Question 3 asks the students to compare the results of question 2a with those from 2b. They will typically find that the actual number of heads and tails in each question does not converge toward the same number but will on average preserve the difference of 10, but the percentage of heads and tails will converge towards 50%. So for question 2a, the results might be 30 heads and 20 tails and the percentages would be 60% heads and 40% tails. For question 2b the results might be 230 heads and 220 tails which would be 51.1% and 48.8%. This illustrates that the percentage of heads and tails gets closer to 50%, but the actual number of heads and tails does not. Therefore betting on tails neither helps or hurts ones chance of winning.

The key concepts here are:

1. that random chance is not self correcting
2. a difference in heads and tail is not corrected, but becomes less noticeable (washed out) by subsequent flips
3. any betting strategy that depends on the idea that random chance will even things up will not work.

The idea that differences get less noticeable as you increase the sample is also illustrated in Student Resource 6: Coin Flips. After 14 flips, a difference of 10 more heads than tails looks really large, but after 414 flips a difference of 10 heads is hard to see at all. The fact that the difference gets less and less noticeable leads to the incorrect belief that random chance corrects itself.

Teacher Resource 3 P. 21

DISCUSSION

Engage the students in a discussion about the unpredictability of the outcomes of coin and dice experiments. The concept of "independence" (in other words, coins and dice do not have memories) from one trial to the next is a critical learning.

During the discussion teachers may wish to use an overhead of Student Resource 5: Recognition of Patterns, to emphasize that human beings naturally seek patterns where none are intended. (Reveal "V V V" and the students will likely say that these are 3 Vs. Reveal the two Vs in close proximity and they might call this a W. Reveal the three rotated Vs and they will likely call this a triangle. Students will likely see pattern where none is intended.)

Another illusion is shown on the next page where the illusion of a triangle is so strong that it actually look brighter than white space on the rest of the page. These images illustrate how good humans are at finding patterns. This pattern recognition skill however can be a problem when we see patterns in random chance. Pattern recognition is a skill. The problem is that our brains are set up to find patterns; not to determine if they are real or not. If we saw a pattern of black and yellow stripes in a forest, are brain might conclude that it is a tiger. There is a survival value to assuming it's a tiger and seeking safety. There is no survival value in deciding testing the pattern to determine if it is a real tiger or just some random stripes pattern. In addition, pure random chance is quite rare outside of a casino. The weather for example, is hard to predict, but there are predictable aspects of weather (e.g., dark clouds mean rain; a bright sun means warmth). Therefore our brain is biased to assume that any pattern is real. This bias help us survive, but can also lead to the belief in superstitions, astrology, conspiracy theories, unconfirmed anecdotal reports, urban legends, and folk wisdom. When a person is gambling and experiencing randomly generated patterns, this same tendency in our brains, may lead us into thinking that the game can be beaten.